

Do Inventory Forward Buys Make Sense?

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Abstract

Procuring commodities is difficult due to the fluctuating purchase prices intrinsic in commodities. These price fluctuations can allow a firm to benefit from buying for future demand in addition to current demand when prices are low. Golabi (1985) proposes a method to determine how many, if any, periods to forward buy. However, the commodity price variability must meet a minimum criterion for forward buys to make sense. This criterion along with publicly available historical price indices can give management and indication regarding if it is worth investigating forward buys for inventory replenishment.

Key words: Procurement; Commodities; Forward Buys; Golabi; Inventory

1.0 Introduction

Purchasing commodities is complex because the purchase cost in the future is uncertain. Prices may increase or decrease in a short amount of time. Golabi (1985) proposes a method to determine cost thresholds, where if the current realized cost to buy crosses a threshold, forward buying should occur to minimize expect cost. His method assumes deterministic demand and a known distribution of commodity costs. We outline Golabi's method using a simple example in section 2.1.

Practically speaking, the future distribution of commodity costs can be estimated from historical price variability using commonly available database listed in section 2.3. The variance and mean of price distributions can be calculated from historical spot prices with the same interval length as a company's buying frequency. From this, managers can asses how often historical prices would have been low enough to warrant forward buying in the past.

A simple method will be shown that determines a lower price threshold. A manager can use this price threshold to determine if forward buys would have been optimal in the past. Given the frequency of realized prices below this price threshold, the probability of future similarly low prices can be calculated. If the price data support that forward buys are likely, then the method outlined by Golabi (1985) can be applied each purchasing period to determine how many periods to forward buy at each buying opportunity.

2.0 Literature Review

2.1 Golabi's Method of Forward Buys

Golabi (1985) proposes a method whereby material for future periods is bought as long as the marginal cost is less than the marginal savings. His recursive heuristic yields a series of decreasing thresholds corresponding to the number of periods to buy forward. Ordering prices in each period are random with a known distribution. Magirou (1987) comments on the similarity between his 1982 paper and Golabi's 1985 work. Golabi's equation accounts for the probability that the next period price will be less than the current price plus the benefit of locking in the prior price minus the holding costs for one period. Equation 1 below is the corrected equation (9) from Golabi's paper that specifies the next price point such that forward buying $n+1$ periods is optimal. A_n is the threshold price per unit such that buying n periods ahead is optimal. If the current purchase price is less than or equal to A_n , then it is optimal to buy for the current period and n periods ahead. Let x be the purchase price and $F(x)$ is the known cumulative price distribution for each period (in equation 1 below $dF(x)$ is equivalent to $f(x)dx$ the probability density function). h is the cost to hold one unit of stock for one period. A_0 is the highest possible purchase price since Golabi assumes all demand must be met for the current period (period 0). Each additional threshold price is computed according to (1) below

$$A_{n+1} = \int_0^{A_n} x dF(x) + \int_{A_n}^{\infty} A_n dF(x) - h \quad (1)$$

Given the probability that the price falls in the future, the first integral in (1) is the opportunity cost of buying in the current period versus buying at a possibly lower cost

next period. The second integral is the benefit of locking the A_n purchase cost. The final term is the holding cost for buying inventory in period n for use in period $n+1$. We will now illustrate this heuristic with stationary, uniform price distributions. Assume that the price at any buying opportunity can be \$25, \$50, \$75 or \$100 – each with equal probability. Assume the holding cost for one period is \$5.

Since we must buy to cover demand in the current period (0), A_0 equals the highest possible price. Thus $A_0 = 100$, the highest possible purchase price of our uniform price distribution.

A_1 is the expected price lower than or equal to A_0 plus the benefit of locking in at the price A_0 minus the one period holding cost h . The first term in equation 1 is the expected price lower than or equal to A_0 is $\$25 * 25\% + \$50 * 25\% + \$75 * 25\% + \$100 * 25\% = \$62.50$. The second term in equation 1 is the expected benefit of locking in the price of A_0 . This second term is \$0 since the price cannot go higher than \$100. We also need to subtract the one period holding cost (\$5 in this example) if we buy now for the next period. $A_1 = \$62.50 + \$0 - \$5 = \57.50 . If the current purchase price is \$57.50 or lower, we should buy for the current period demand plus the demand for the next period.

Likewise, $A_2 = \$18.75 + \$28.75 - \$5 = \42.50 , $A_3 = \$6.25 + \$31.88 - \$5 = \33.13 , $A_4 = \$6.25 + \$24.84 - \$5 = \26.09 , and $A_5 = \$19.57$ which is below the possible price range for the distribution so we can be certain that we will never buy for more than four periods in advance.

This method seeks to answer how many periods in advance should we buy to satisfy all predicted demand and to minimize total expect costs. Given our price

distribution, Table 1 shows the current purchasing prices that make sense for forward buying.

Table 1: Price Thresholds to Forward Buy

		Calculated					
		Value	\$ 25.00	\$ 50.00	\$75.00	\$ 100.00	No Discount
A0	\$	100.00	Yes	Yes	Yes	Yes	
A1	\$	57.50	Yes	Yes			
A2	\$	42.50	Yes				
A3	\$	33.13	Yes				
A4	\$	26.09	Yes				

2.2 Historical Commodity Prices

The historical prices for can be found from a variety of sources including American Metal Market, LLC (www.amm.com) for metals, Random Lengths (www.randomlengths.com) for forest products, and The American Economagic (www.economagic.com) for many different commodities. The American Economagic is a website providing economic time series data for classroom use. Commodity prices on this site are under Producer Price Index by Product. The database contains Seasonally Adjusted (SA) prices and Non-Seasonally Adjusted (NSA) prices in U.S. Dollars for these categories of products:

- * 01 Farm products: SA, NSA
- * 02 Processed foods and feeds: SA, NSA
- * 03 Textile products and apparel: SA, NSA
- * 04 Hides, skins, leather, and related products: SA, NSA
- * 05 Fuels and related products and power: SA, NSA
- * 06 Chemicals and allied products: SA, NSA
- * 07 Rubber and plastic products: SA, NSA
- * 08 Lumber and wood products: SA, NSA
- * 09 Pulp, paper, and allied products: SA, NSA
- * 10 Metals and metal products: SA, NSA
- * 11 Machinery and equipment: SA, NSA
- * 12 Furniture and household durables: SA, NSA
- * 13 Nonmetallic mineral products: SA, NSA
- * 14 Transportation equipment: SA, NSA
- * 15 Miscellaneous products: SA, NSA

Under each category are detailed products. For example, under 08 Lumber and wood products, the following products are found:

- * **08 Lumber and wood products: SA, NSA**
- * 081 Lumber: SA(more), NSA(more)
- * 082 Millwork: SA(more), NSA(more)
- * 083 Plywood: SA(more), NSA(more)
- * 084 Other wood products: SA(more), NSA(more)
- * 085 Logs, bolts, timber and pulpwood: SA(more), NSA(more)
- * 086 Prefabricated wood buildings and components: SA(more), NSA(more)
- * 087 Treated wood and contract wood preserving: SA(more), NSA(more)

For more information on these databases, see Manikas (2007). From the free online database Economagic, Table 2 shows the monthly spot price data for plywood from 1996 to 2004.

Table 2: Monthly Spot Prices for Plywood: 1996 - 2004

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1996	\$ 154.00	\$ 154.20	\$ 152.10	\$ 151.60	\$ 158.30	\$ 155.80	\$ 154.00	\$ 157.40	\$ 166.70	\$ 160.10	\$ 157.90	\$ 155.30
1997	\$ 154.50	\$ 158.80	\$ 163.60	\$ 159.10	\$ 158.20	\$ 162.70	\$ 162.60	\$ 162.60	\$ 162.60	\$ 156.00	\$ 157.90	\$ 153.60
1998	\$ 152.90	\$ 153.30	\$ 151.30	\$ 152.40	\$ 151.40	\$ 151.10	\$ 159.10	\$ 167.60	\$ 170.00	\$ 156.70	\$ 160.90	\$ 161.10
1999	\$ 161.60	\$ 169.30	\$ 173.00	\$ 170.20	\$ 179.10	\$ 196.90	\$ 208.30	\$ 202.10	\$ 177.80	\$ 158.70	\$ 159.50	\$ 160.80
2000	\$ 162.00	\$ 162.20	\$ 166.00	\$ 167.20	\$ 156.60	\$ 155.30	\$ 154.00	\$ 152.60	\$ 156.40	\$ 155.10	\$ 152.60	\$ 150.90
2001	\$ 147.00	\$ 145.60	\$ 148.00	\$ 147.10	\$ 165.10	\$ 166.50	\$ 156.80	\$ 163.80	\$ 161.20	\$ 151.60	\$ 150.30	\$ 148.00
2002	\$ 148.30	\$ 151.50	\$ 160.30	\$ 159.30	\$ 152.30	\$ 153.30	\$ 150.60	\$ 152.40	\$ 149.70	\$ 149.30	\$ 147.30	\$ 146.30
2003	\$ 145.90	\$ 147.20	\$ 146.00	\$ 145.60	\$ 145.40	\$ 148.90	\$ 162.30	\$ 166.30	\$ 193.90	\$ 203.90	\$ 206.30	\$ 192.10
2004	\$ 174.40	\$ 202.50	\$ 218.50	\$ 222.60	\$ 224.40	\$ 200.80	\$ 178.60	\$ 203.10	\$ 208.30	\$ 189.00	\$ 173.90	\$ 185.50
2005	\$ 187.00	\$ 191.50	\$ 187.80	\$ 183.80	\$ 174.00	\$ 186.30	\$ 181.90	\$ 176.80	\$ 200.10	\$ 211.60	\$ 180.70	\$ 180.20

Figure 1 shows a graphical representation of the data.

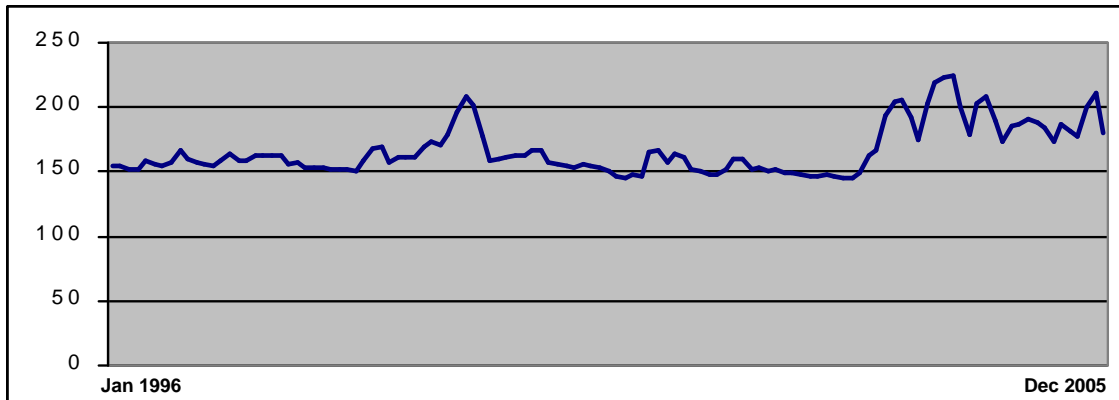


Figure 1: Historical Plywood Prices 1996-2005

The plywood prices in Table 2 had a maximum one-month price swing of 16.6%. A holding costs of 20% per year is equivalent to a monthly holding cost of 1.66%. In this case, 50.4% of the price movements in Table 2 deviate more than 1.66% from month to month. These fluctuations can be seen graphically in Figure 1. For an annual holding cost of 12%, the price swings of the plywood data in Table 2 are beyond the holding costs 65% of the time. These significant fluctuations indicate that forward buying may make sense, and therefore the threshold calculation outlined in section 5 should be used. Certainly, the assumption of non-speculative buying would not make sense for this plywood data.

2.4 Random Walk Costs

Some commodity prices may reasonably be modeled as a random walk [Working (1934); Kendall (1953); Roberts (1959)]. These authors note that a random walk may accurately portray the increasing uncertainty in future prices each period forward. Moinzadeh (1997) investigates price discounts (deals) at random points in time with

negligible lead times and exponentially distributed times between deals. Our random walk approach differs because 1) the price may increase or decrease, and 2) the discount amount is not known, just the distribution. Grubbström and Kingman (2004) use a net present value model to replace Economic Order Quantity (EOQ) decisions for known future price increases that are announced in advance.

The price distribution F_i would then be as shown in Figure 2 where i is the period number from the current period forward. c is the cost per unit to buy now (period 0). Assume that each period forward there is a $1/3$ chance each of 1) the cost staying the same, 2) the cost going up by c or 3) the cost going down by c .

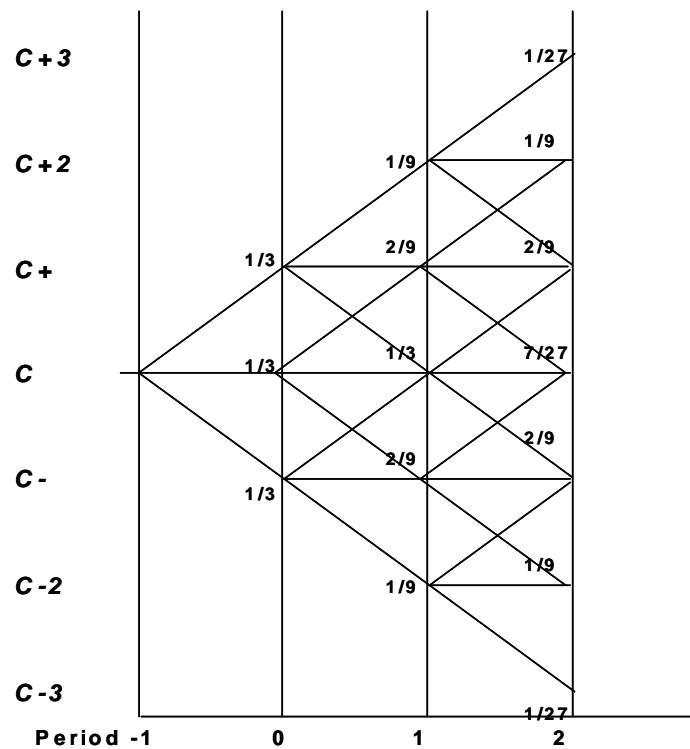


Figure 2: Random Walk Probabilities

We assume that we have observed the last cost c , so the distribution for period 0 is from c^- to c^+ . A_0 is therefore c^+ by definition. Rewriting (1) for a random walk price distribution, the cost threshold to forward buy one period becomes (2) below.

$$A_1 = \frac{1}{3}c + \frac{2}{9}(c - \lambda) + \frac{1}{9}(c - 2\lambda) + \frac{1}{3}A_0 - h \quad (2)$$

Plugging in the known value for A_0 yields (3).

$$A_1 = c - \frac{1}{9}\lambda - h \quad (3)$$

Even if h was 0, A_1 would still be a value less than c since $\lambda > 0$ by definition. This knowledge will allow us to determine the correct probabilities for the next threshold A_2 . The current cost distribution is c^- or higher, therefore we can set (3) to be greater than or equal to this lowest possible purchase cost to find a threshold for the holding cost h . Substituting in the known value of A_0 , and solving for h , we get the following for constraint for h :

$$h \leq \frac{8}{9}\lambda \quad (4)$$

Regardless of the last observed cost c , holding costs for one period must be no more than $8/9^{\text{th}}$ of the random walk step, otherwise no forward buying should ever occur.

Similarly, we can derive the equation A_2 in (5), then solve it in (6).

$$A_2 = \frac{1}{27}(c - 3\lambda) + \frac{1}{9}(c - 2\lambda) + \frac{2}{9}(c - \lambda) + \frac{12}{27}A_1 - h \quad (5)$$

$$A_2 = c - \frac{152}{243}\lambda - \frac{44}{27}h \quad (6)$$

Setting this to be less than or equal to c allows us to solve for the constraint on h in (7) below.

$$h \leq \frac{91}{396}\lambda \quad (7)$$

We can derive an equation for A_3 as shown in (8), then solve it and its holding cost threshold as shown in (9) and (10).

$$A_3 = \frac{1}{81}(c - 4\lambda) + \frac{4}{81}(c - 3\lambda) + \frac{10}{81}(c - 2\lambda) + \frac{16}{81}(c - \lambda) + \frac{50}{81}A_2 - h \quad (8)$$

$$A_3 = c - \frac{20236}{19683}\lambda - \frac{4387}{2187}h \quad (9)$$

$$h \leq -\frac{553}{19683}\lambda \quad (10)$$

Notice that the holding cost would have to be negative in (10) since λ is positive by definition. Therefore, it is never recommended to buy more than two periods forward under a random walk. If the distribution in the current period is assumed to be the point cost c , then no forward buying is ever recommended as was shown in Manikas (2007).

3.0 Non-Random Walk Distributed Prices

Given the availability of historical commodity spot prices shown in section 2.3, it is certainly possible that the prices do not exhibit random walk behavior. If prices are normally or uniformly distributed, then the average historical price can be calculated easily. The necessary lower price threshold can be established to indicate if forward buying would have been optimal in the past. Assuming these past prices reflect the future price realizations, a manager can have a degree of confidence that if the lower price threshold was met in the past that it may occur in the future. Therefore, forward buys may be optimal for this particular item in the future and the Golabi thresholds should be calculated for this particular commodity product.

To find the lower threshold, the average historical price should be calculated. The *low* threshold is shown in (11) below, where h is the holding cost percentage per period. For example, if the annual holding cost is 20%, and procurement is done once every month, then h is $20\% / 12$ or 1.66%. If the average purchase price is \$100, then *low* is calculated to be $\$100 / (1 + .0166) = .9836$. Therefore, if the historical price was ever \$98.36 or lower than forward buying would have been optimal in the past for the period where the historical price was at or below this *low* threshold. The historical spot prices can be searched to see if the price was ever equal to or lower than the *low* price.

$$low = \frac{average}{1 + h} \quad (11)$$

3.1 Uniformly Distributed Prices

If historical prices are uniformly distributed, then the likelihood that forward buying will make sense is:

a = lower bound of uniform distribution

b = upper bound of uniform distribution

p = probability that forward buying would be optimal is calculated below in (12).

$$p = \frac{1}{(b-a)} \left(\frac{(b+a)}{2} - a \right) \quad (12)$$

A manager can use this probability to determine how likely forward buys would have been in the past. The lower (higher) p is, the less (more) likely forward buys would have been optimal to minimize expected costs. Clearly, the historical data can be searched to arrive at an exact number of periods that the actual price was at or below the *low* value to given an exact probability.

3.2 Normally Distributed Prices

If prices appear to be normally distributed, then the mean and standard deviation can be calculated and used in (13) below to compute p , the probability that forward buying would be optimal. We show the Microsoft Excel function for this calculation since that spreadsheet application is so widely available in business and the math for computing probabilities for a normal distribution is built into that office productivity software application.

$$p = \text{normdist} (\text{low} , \text{mean} , \text{standard deviation} , \text{TRUE}) \quad (13)$$

This probability can be used by management to assess the probability that the historical price would have been below the *low* threshold indicating that forward buys would have been optimal in the past. Again, the exact number of times the historical price was at or below *low* can be counted instead of using the calculated probability in (13). Assuming the future will have similar price variability to the past, this probability gives managers a good indication if forward buying might be beneficial for their situation.

4.0 Conclusions and Discussion

If a random walk fits the commodity data in question, then the manager can conclude that forward buying of more than two periods is never optimal. Even with zero holding cost, the expected savings equal the expected costs, therefore the added uncertainty of buying further in the future is a disincentive to forward buy. Since each commodity is different, depending on the time period and buckets analyzed, whether or not a random walk fits the price data can only be ascertained on a case by case basis given the particular commodity, date range, and frequency.

For uniform or normally distributed price distribution based on past data, the probabilities that the price was ever low enough to make forward buys expected to optimize costs were outlined in equations 3.1 and 3.2 respectively. A manager needs to use this information regarding the optimality of forward buys in the past to decide if forward buys are worth investigating for future procurement. If the probability is sufficiently high, and the future price distributions are assumed similar to historical price

distributions, then the method outlined in Golabi (1985) may be applied to determine the optimal number of periods to forward buy given the current purchasing cost and assumed future price distribution.

Krishna (1994) uses a Weibull distribution to model time between deals (low prices) to build on Golabi's method. Similarly, for any well behaved purchase cost function, the applicability of forward buys can be ascertained beyond those functions we evaluated in section 3.

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