EVALUATING FORECASTS OF RATES OF RETURN: GEOMETRIC AND ARITHMETIC MEANS DUAL IT OUT

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ABSTRACT

Debate continues on the proper approach for forecasting rates of return. Generally, past performance of similar risk-class asset returns serves as the determinant for expected future returns. These forecasts inform strategies applicable to endowments, retirement and trust funds, and generally to any investment. Traditionally, arithmetic averages (means) have served as the historical metric. Argumentatively, using a geometric mean of historical performance may provide more reliable measure of expected returns. A dual approach blends the traditional methods using a weighted average of the two to overcome the inherent biases of the individual measures. This paper evaluates the three forecasting methods against realized returns to determine the efficacy of each.

INTRODUCTION

Calculating actual returns on investments relies on an arithmetic mean which generates accurate and unambiguous results. Using arithmetic means of historical returns as a forecast for the future, however, can be upwardly biased depending on the forecast period. Longer-termed forecasts benefit from the use of geometric means, however in the shorter time horizons, a downward bias may occur. Blume [1] explores the biases of both estimation methods and offers a weighted average approach that combines the two methods in a way that counteracts the inherent bias in each.

Work by others in finance that extends beyond common finance texts have explored this condition [2, 3, 4, 5] and interestingly, other disciplines face the same dilemma of choosing an appropriate metric [7, 8].

The purpose of this paper is to evaluate the efficacy of estimates using arithmetic, geometric, and averaged means when applied to specific data sets of market returns. In particular, data from the Dow Jones Industrial Average (DJIA), Standard and Poor's 500 index (S & P 500), and United States treasury securities (notes and bills) are of interest to continuing research. Evaluating the accuracy of forecasts built on historical data for varying investment time horizons (1-, 5-, 10-, and 20-years) and errors can provide some guidance to future research in estimating rates of return. Of special interest to future research is the impact of varying investment strategies on realized returns when those strategies are dependent upon mean return forecasts.

DATA AND METHOD

Daily closing values for the various securities and indices will be used as the basis for forecast returns within each class of assets. For example, data from the DJIA spanning 1928 through 2007 will be included. Where daily values are unavailable for other assets, monthly or quarterly values will be employed.

Where periodic returns are less than yearly, the annual returns using the arithmetic mean (A^t) for time period "t" and periodic returns for the i^{th} period are given by:

$$A^t = \frac{1}{n}\sum_{i=1}^{n}(1+r_i) \quad (1)$$

Similarly, annual returns for less than yearly periodic rates of return using the geometric mean for time period "t" (G^t) are given by

$$G^t = \sqrt[n]{\prod_{i=1}^{n}(1+r_i)} - 1 \quad (2)$$

Equations (1) and (2) can be applied to multi-year calculations, as well, such that the arithmetic mean of a 10-year historical data set simply sums the to yearly annual returns and divides by 10. Similarly, the same 10 years of data can generate a geometric mean using (2) where n = 1 to 10.

Blume's [1] formula, as adapted by Ross, *et al.* [6], suggests using a weighted average of the geometric and arithmetic means of annual values to generate an expected return (R^t), where the weighting factors are determined as the proportion of the forecast period to the historical review period, such that,

$$R^t = \left\{ \left[\frac{T-1}{N-1} \right] * G^t + \left[\frac{N-T}{N-1} \right] * A^t \right\} \qquad (3)$$

where T is the number of periods of historical data used and N is the forecast horizon [6].

Calculating forecasts using the three methods for the various investment horizons will be compared against the actual returns for those periods. Tests for differences and significance should provide insight into the efficiency of each.

RESULTS

To be determined.

CONCLUSIONS

To be determined.

REFERENCES

[1] Blume, Marshall E. (1974). "Unbiased estimators of long-run expected rates of return." Journal of the American Statistical Association 69 (347), 634-638.

[2] Cooper, Ian. (1996). "Arithmetic versus geometric mean estimators: Setting discount rates for capital budgeting." European Financial Management 2 (2), 157–167.

[3] Brodsky, D.A. (1982). "Arithmetic versus geometric effective exchange rates." Review of World Economics 118 (3), 546-562.

[4] Bruner, R.F., Eades, K.M., Harris, R.S, Higgins, R.C. (1998). "Best practices in estimating the cost of capital: Survey and synthesis." Financial Practice and Education, Spring/Summer.

[5] Jacquier, E., Kane, A., Marcus, A.J. (2003) "Geometric or arithmetic mean: A reconsideration." Financial Analysts Journal.

[6] Ross, Westerfield, and Jordan. (2008). Fundamentals of Corporate Finance, 8th edition. McGraw-Hill Companies.

[7] Seixas, N.S, Robins, T.G., Moulton, L.H. (1988). "The use of geometric and arithmetic mean exposures in occupational epidemiology." American Journal of Industrial Medicine 14(4), 465-77.

[8] Smothers, C.D., Sun, F., and Dayton, A.D. (1999). "Comparison of arithmetic and geometric means as measures of a central tendency in cattle nematode populations." Veterinary Parasitology 81 (3), 211-224.