DETERMINING THE VALUE OF A VOTE IN THE UNITED STATES UNDER THE ELECTORAL COLLEGE VOTING SYSTEM USING LINEAR PROGRAMMING

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ABSTRACT

The Electoral College system in the United States allows for a leader to be elected to the position of president without the support of the majority of the population by a large margin. In this paper we derive the minimum percentage of registered voters required to elect a president by creating a binary integer programming problem to represent the minimum number of registered voters to win the Electoral College. To find this minimum number, we make some reasonable assumptions to cut down on run time and then apply an algorithm that we created to exhaustively test every remaining possibility. To confirm this result, we also apply Balas Algorithm to the problem. We compare this minimum number of voters against the whole population to see the magnitude of the difference. We found that only 21.91% of the registered voters are required to win a majority vote in the current Electoral College system. This represents less than 10% of the total U.S. population.

Keywords
Electoral College system; Integer Programming; Balas Algorithm

INTRODUCTION

The Electoral College is a system that was uniquely designed by the founding fathers for the United States intent on overcoming certain difficulties faced when trying to equally represent a large group of dispersed people with limited means of communication and transportation. Although it was also originally created intent on preventing a party system from emerging in the U.S, the college is now a major proponent of the strong bi-partisan system it has. While the college did achieve most of its intended goals and seemed to adequately represent the targeted people, it also allows for a very interesting extremity. A candidate in this system can be elected to the status of president by a majority of the college votes while simultaneously holding a minority of the popular votes. Knowing this, one might raise questions regarding its place in today's voting system. Is a system that was designed with very specific obstacles in mind no longer satisfying its ultimate goal, to represent the will of the people, in the most effective way possible?

To derive just how inadequately the Electoral College system can represent the people, we model the Electoral College with a binary integer programming problem. Through this model we are able to minimize the number of citizen's votes while maintaining a majority vote in the Electoral College.

THE ELECTORAL COLLAGE

Although it has since been slightly modified, the founding fathers created the Electoral College system that we use today. In its creation, they faced interesting problems in regards to how to elect a national leader. How can the citizens of thirteen states of various sizes and interests be encouraged to elect a single candidate with a majority vote to represent them all? Emphasizing this dilemma was a lack of communication and transportation among its citizens, especially since the act of campaigning for a public official position was frowned upon and because of this, no one should be actively seeking recognition in hopes of achieving the position of president. Furthermore, the possibility of a growing party system was particularly scary to the founding fathers as they had much knowledge of the corrupt party system in Britain and were hoping to avoid the possibility of such a system becoming established in their new homeland. To combat these issues, it was decided that a group of politically informed individuals, including two senators and at least one (based on state population) non-government affiliated representative, would vote for a national leader in place of, what we now know as, a popular vote. Kimberling (1992) says, “The original idea was for the most knowledgeable and informed individuals from each State to select the president solely on merit and without regard to State of origin or political party” (p. 2) This system of voting served its purpose well in every aspect except that the grouping of people seeking political clout by compromising some values for the sake of others seems to be an inevitability and, even in this system, a party system emerged.

The Electoral College was originally created intent on discouraging a party system from forming, however the opportunities a party system brings for groups with similar ideas to increase control over the nation must have been too attractive as parties were formed anyway. According to Duverger’s law, the plurality rule system, in which the candidate with the most votes receives all of the support, that the U.S. has in all states, excluding Maine and Nebraska, encourages a two-party system over a multiparty (more than two parties) system by making it extremely difficult for a third party to receive enough political support to earn even a single electoral vote. As a result, voters are more likely to vote for one of the two major candidates since voting for a third-party would most likely be the equivalent of having no vote at all. According to Grofman (2009), “This behavior can induce a self-fulfilling prophesy whereby minor party candidates rarely achieve an aura of viability that permits them to effectively compete alongside their major party opposition” (p. 65).

INTEGER PROGRAMMING

An integer programming problem is an optimization problem wherein the variable of the system represents only integer possibilities. This sort of problem can be extremely useful when representing a lot of real world scenarios that take on only discrete possibilities. A binary integer programming problem is a special kind of integer programming problem in that the domain of the variable is trimmed down to only two possibilities: 0 or 1. This sort of problem is useful when representing something that has only two possibilities, such as 'on' or 'off', or 'for' or 'against'. A binary integer programming problem is the kind of problem that we use to solve the proposed question: What is the minimum number of voters required to award a candidate a majority of the electoral votes?

State	Voters per Electoral Vote	Electoral Votes	State	Voters per Electoral Vote	Electoral Votes	State	Voters per Electoral Vote	Electoral Votes
WY	79,666.67	3	ME	194,750.00	4	KY	264,125.00	8
DC	98,333.33	3	AR	209,333.33	6	AZ	266,727.27	11
AK	105,666.67	3	KS	224,833.33	6	VA	272,769.23	13
HI	116,500.00	4	OK	229,000.00	7	MN	275,600.00	10
VT	118,666.67	3	CT	234,428.57	7	FL	275,655.17	29
ND	120,333.33	3	AL	247,111.11	9	WA	280,583.33	12
RI	127,500.00	4	TX	249,815.79	38	OR	286,428.57	7
SD	135,333.33	3	MD	250,900.00	10	NY	289,482.76	29
DE	139,333.33	3	CA	252,072.73	55	WI	290,800.00	10
NM	149,200.00	5	GA	254,750.00	16	IL	291,150.00	20
NE	160,800.00	5	MS	255,333.33	6	MA	293,636.36	11
MT	163,666.67	3	CO	255,444.44	9	NC	297,000.00	15
NV	166,000.00	6	SC	256,000.00	9	MO	301,300.00	10
ID	166,500.00	4	IA	257,833.33	6	PA	301,550.00	20
NH	166,500.00	4	IN	258,000.00	11	LA	303,125.00	8
WV	176,600.00	5	TN	259,454.55	11	OH	311,166.67	18
UT	182,500.00	6	NJ	261,142.86	14	MI	320,437.50	16

Figure 1. The ordered weights of voters per electoral vote for each state.

Although the opinion of whether a ‘plurality rules’ system in the U.S. is better than any proposed alternative can be debated, what we’re interested in is the extreme possibility case in the electoral college system together with the plurality system employed by most states. The Electoral College enables a U.S. President to be elected by a majority vote in the college while not having a majority of the popular vote. This is a result of not being able to represent many groups of people’s votes with a single vote equally. Because of the process of determining how many electoral votes each state should get, “... the apportionment of electoral votes always over-represents some states and under-represents others.” according to Edwards III (2004, p. 2). As can be seen from the table, the number of voters per electoral vote has an outstanding range from around 80 thousand voters per electoral vote in Wyoming to 320 thousand voters per electoral vote in Michigan. This data was calculated based off of the number of registered voters per state at the time of the 2010 census. To acquire all of the votes from a particular state, a candidate needs to only obtain more votes than any other candidate in that state. If there are only two candidates, then the winning candidate only needs more than 50% of the citizen's votes in a particular state to receive 100% of the state’s support through electoral votes. If there are more than two candidates, the percentage can be much less (33% for three, 25% for four, etc), but since the U.S. is a bi-partisan nation, we will make the safe assumption that there are only two candidates in our problem. Furthermore, more than two candidates serve only to strengthen the point of the paper by allowing a candidate to be elected president with even fewer votes.

As it is clearly evident by history and by the misrepresentations described above, it is possible to be elected president by majority Electoral College vote while maintaining a minority popular vote, but how extreme can this situation become? To derive this, we must represent the situation with a system of weighted sums in which the number of citizen's votes and electoral votes is either counted or not counted. We must find the minimum number of citizen's votes such that the sum of electoral votes is greater than 270. The model of this system is known as a binary integer programming problem and can be written as follows:

Minimize $$Z=\sum_{i=1}^{51} b_i x_i$$

Subject to: $$\sum_{i=1}^{51} a_i x_i \geq 270$$

$$x \in \{0,1\}$$

where a_i is the number of electoral votes per state, b_i is the minimum number of citizen's votes required to win the electoral vote for the state, and x_i is a binary variable, 1 or 0, depending on whether the state wins or loses the electoral college, respectively. The function to be minimized is known as the objective function and should be ordered such that b_i is less than or equal to b_{i+1}.

OUR SOLUTION

We found a solution to this problem using two methods. First, we derived a means of solving it that will test every possible value. Secondly, by using the Balas Algorithm, we can find a solution in a much shorter time. We describe and implement both methods.

The Exhaustive Method

To solve this problem, we initially wrote an algorithm that exhaustively tests every possible combination of states being for or not for a candidate, verified that the value of electoral votes is equal to or greater than 270, and compared the value of the objective function with any previous findings, replacing it if the new value was lower. The run-time of the algorithm was found to be approximately ten minutes for all combinations of two possibilities on 20 variables. To do all 51 variables would take 231 times longer, which would take about 41 thousand years. In order to maximize the probability of finding the absolute minimum in the range of the number of possibilities we are able to try in a timely manner, it makes sense to assume that the states with the least voters per electoral vote will have to be for the candidate and the states with the most voters per electoral vote will be have to be against the candidate. Following this logic, we let the first 21 states be proponents for the candidate by the minimum number of voters possible and the last 10 states be against the candidate. Algorithm A.1 was used to find the solution in this manner.

The Balas Algorithm Method

While the above solution has a high probability of being correct, having excluded some solutions from being tested, the possibility that we missed the minimal solution still remains. By following the Balas Algorithm, we can find the same optimal solution by testing possibilities in a much shorter time than the exhaustive method is capable of. In this algorithm, we take full advantage of our two well determined constraints. Our program that applies the Balas Algorithm took less than 1 minute to complete and found the same minimal solution. We implemented algorithm A.2 to take advantage of this method.

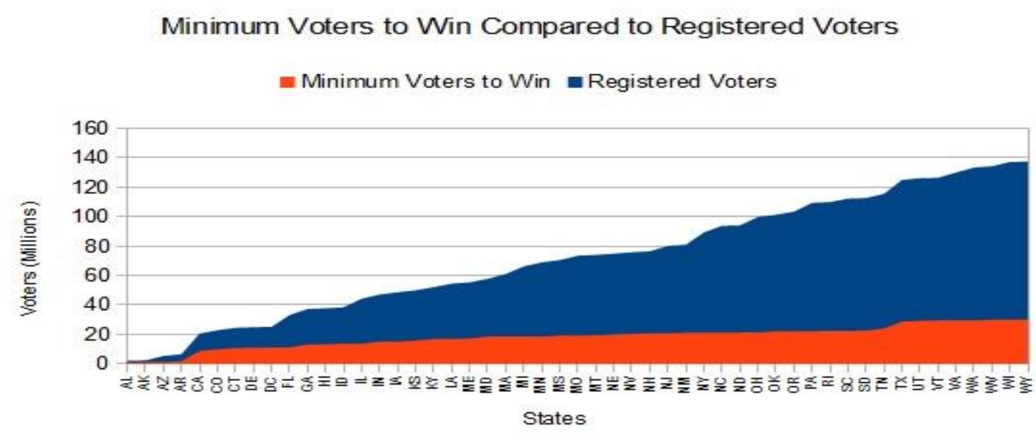

Figure 2. A graphical display of the accumulation of voters to win compared with registered voters.

The figure above is a graphical representation of the solution. We found that the sum of the minimum voters to win the Electoral College by majority vote is 21.91% of the sum of the registered voters in the U.S. and only 9.77% of the total population of the U.S. We also found that in a proportional representation system where a percent of each state's electoral votes would go toward separate candidates based on the percentage of voters in the state that voted each way, the minimum percentage of votes to win the Electoral College by majority increases to a little more than 39%.

CONCLUSIONS

Although at first glance the proportional representation system seems like a very appealing and obvious alternative to our current system, according to Duverger's law, it would lead to a multi-party system which would clearly increase the probability of candidates winning the electoral college by a plurality vote (more than the rest, but less than 50%) more often than they do now and by a much greater split. It is, however, significant to note that it would require five almost equally competitive candidates to push the minimum winning proportional representation system's plurality percentage down to the minimum winning U.S. plurality system's majority percentage.

APPENDIX A. ALGORITHMS

The algorithms below were used when finding the solution to the proposed question.

A.1. Exhaustive Algorithm

1. *LET S be an array of arrays such that the inner arrays are comprised of the state name, the total of registered voters in the state, the number of votes required to win the electoral college, and the number of electoral votes assigned to the state.*
2. *FOR each element in S;*

    Add up the 'Registered voters' sub-element.
3. *SORT S by number of votes required to win the college in the state from least to greatest.*
4. *DEFINE the constraint function:*
5. *DEFINE the objective function:*
6. *LET w = 100% be the minimum number of voters required to win the Electoral College.*
7. *FOR i from 0 to 251 (base 2);*

    Input the characters of i into an array, B, in reverse order.

    WHILE the length of B is less than 51;

        Add 0's to the array.

    Evaluate constraint function at B.

    IF constraint function at B is +270;

        Evaluate objective function at B (Percentage to Registered Voters);

        IF objective function at b is less than w;

            w = objective function at B
8. *RETURN w*

A.2. Balas Algorithm

1. *LET S be an array of arrays such that the inner arrays are comprised of the state name, the total of registered voters in the state, the number of votes required to win the electoral college, and the number of electoral votes assigned to the state.*
2. *FOR each element in S;*

    Add up the 'Registered voters' sub-element.
3. *SORT S by number of votes required to win the college in the state from least to greatest.*
4. *DEFINE the constraint functions:*
5. *DEFINE the objective function:*
6. *LET w = 100% be the minimum number of voters required to win the Electoral College.*
7. *LET A = [1] be an array.*
8. *DEFINE a function F that appends 0's to A till the length of A is 51.*

9. EVALUATE the constraint functions at F(A)

10. IF F(A) satisfies all constraints:

IF the objective function at F(A) is less than w:

w = objective function at F(A) divided by total voters.

11. IF the constraint function evaluated at F(A) is less than 270:

Add a 0 to the beginning of A and repeat 9 through 11.

Add a 1 to the beginning of A and repeat 9 through 11.

12. RETURN w

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